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Vehicle Control by Flatness

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Abstract

This study is aimed at looking into the flatness control of the vehicle detailing adopted mechanisms and approaches in order to be able to control this system in the presence of disturbances and to solve problems encountered during its functioning. A trajectory tracking controller based on differential flatness is presented for a nonlinear vehicle model. Selected results of numerical simulations are shown. At the end of this work, a comparative study between the real moving and the desired one has been presented.

Keywords

vehicle, flatness control, path planning, path tracking.

1 Introduction

Vehicles driving at high speeds have received substantial research attention recently, as highlighted by the DARPA Grand Challenge competitions[1],[2]. This paper considers the problem of trajectory tracking control near the limits of tire-road friction for robotic vehicles with front wheel steering. A challenge of vehicle control near the limits of friction is the nonlinear behavior of the tire forces and potentially unstable dynamics for large slip angles. Unstable equilibrium points may be present, depending on the vehicle speed, steering angle, and tire properties[3]. A path tracking control algorithm commonly used for robotic vehicles with front wheel steering is pure pursuit[4]. These approaches support a variety of models and performance objectives, though the computational demands of real-time numerical optimization may be prohibitive, particularly for low-cost microcontrollers. Recent research in trajectory tracking control has focused on systems with a property known as differential flatness. A nonlinear system$\dot{x} = f(x, y)$is differentially flat if an output y can be found such that the states x and inputs u can be expressed in terms of y and a finite number of its derivatives[5]. The flatness property was introduced by M. Fliess, J. Lévine, P. Martin and P. Rouchon in 1992 to propose a new strategy to control continuous nonlinear systems with good performances in term of tracking trajectory. At first, the use of this property consists in the definition of an output trajectory allowing the determination of the variables of the flat system. Secondly, it concerns the elaboration of the control in closed- loop allowing, to obtain a stable system giving place to a tracking of a desired trajectory with an error which tends asymptotically towards zero. A benefit of flat systems is that flat outputs can follow arbitrary trajectories y_d provided that the trajectory is sufficiently smooth.
Many scientists have used the application of the differential flatness theory in order to solve problems in relation to the motion of vehicles [6], [7], [8], [9].

This paper presents a trajectory tracking controller for the flat output located at the position of the rear wheel. The position of this point is an advantageous choice of flat output as it can be controlled to track trajectories with finite acceleration. The states and the in puts of the flat system can be expressed in function of the particular out puts and their successive derivatives. We can find a lot of the literature uses linear approaches[10],[11], [12]or approaches of optimal control [13]. A benefit of flat systems is that flat outputs can follow arbitrary trajectories $y_d(t)$ provided that the trajectory is sufficiently smooth. For example, a front-steered bicycle driving in a plane without wheel slip is a flat system whose flat output is the position of the rear wheel[14], which has been exploited for vehicle tracking control [15], though the no-slip assumption restricts its applicability. Another flat output is the position of the Huygens center of oscillation (C.O.) of certain types of rigid body systems, including a vertical take-off and landing aircraft[16], [17]. States related to this point have been identified as flat outputs for a bicycle model with friction forces acting at the front and rear tires, as described below. There exists a point near the front wheels whose lateral body-fixed acceleration is decoupled from the rear lateral tire force. This decoupling was exploited to design a steering controller for the body fixed acceleration at that point[18]. It was later noted that both the point identified by *Ackermann* at the front of the vehicle and a similar point at the rear of the vehicle are centers of oscillation. The body-fixed velocity components at the rear C.O. were identified as flat outputs and a corresponding flatness-based controller defined[19]. The position of the rear C.O. was chosen as the flat output in a trajectory tracking controller by Setlur [20]. Other flat outputs have been considered under the assumption of constant speed and linear tire force models [21],[22].

This paper presents a trajectory tracking controller for the flat output located at the rear C.O. The position of this point is an advantageous choice of flat output as it can be controlled to track trajectories with finite acceleration; however the front C.O. requires an additional degree of smoothness in reference trajectories. We here interested to exploit the concept of the flatness in order to control the system vehicle. This paper is organized as follow: in section 2, we present the dynamic model of the vehicle. In section 3, the vehicle is modeled by a flat system. Section 4 deals with flatness and linearization. Section 5 deals with flatness and trajectories generation. In section 6, we present the flatness and the tracking of trajectory. Finally, section 7 shows experimental results

2 Formulation of the problem

We consider a four wheel vehicle driving without sliding on a horizontal plane. We suppose a point Pwhich its coordinates are(x, y) on the plan$(O, X, Y)$and another point Q respectively the middle of the rear axle and the front axle $(figure1)$. We have $PQ = l$, θ the angle between the vehicle axis and the OX axis and φ the steering angle of the wheels. The conditions without sliding bearing are: $\frac{d\overrightarrow{OP}}{dt}$ is parallel to $\overrightarrow{PQ}$et $\frac{d\overrightarrow{OQ}}{dt}$ is parallel to the front wheels. We note: $u = \frac{d\overrightarrow{OP}}{dt}.\frac{\overrightarrow{PQ}}{PQ}$. The tabular diagram of the vehicle is given by the following figure:

2.1 Model of the vehicle

We consider the dynamic model of the vehicle:

$$\dot{x} = u\cos\theta \quad (1)$$
$$\dot{y} = u\sin\theta \quad (2)$$
$$\dot{\theta} = \frac{u}{l}\tan\theta \quad (3)$$

The implicit form done:

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \quad (4)$$

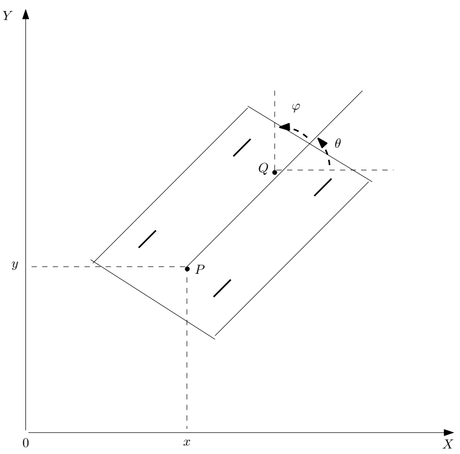

Figure 1: A driving vehicle without sliding

3 Flat systems

Let us consider the nonlinear system described by the following differential equation:

$$\dot{x} = f(x, u) \quad (5)$$

with,$x \in R^n$:the state of the system, and$u \in R^m$:the control of the system. Flatness implies the existence of a vector-valued function h;
ie:

$$z = h(x, u_1, ..., u_1^{(\beta_1)}, ..., u_m, ..., u_m^{(\beta_m)}) \quad (6)$$

where$(z = z_1, ..., z_m)$.The components of x and u are, moreover, given without any integration procedure by the vector-valued functions A and B :

$$x = A(z_1, ..., z_1^{(\alpha_1)}, ..., z_m, ..., z_m^{(\alpha_m)}) \quad (7)$$
$$u = B(z_1, ..., z_1^{(\alpha_1+1)}, ..., z_m, ..., z_m^{(\alpha_m+1)}) \quad (8)$$

Then, to show that a system is flat differentially, it is sufficient to find a flat output and this last has often a physical sense. We can choose the position of the point P denoting by (x, y), and we verify if it presents a flat output, that is to say, verify that all variables and all controls of the system can be expressed in function of this chosen output. The equation (1) and (2) done:

$$\tan\theta = \frac{\dot{y}}{\dot{x}} \quad (9)$$
$$u^2 = \dot{x}^2 + \dot{y}^2 \quad (10)$$

We derive the expression of $\tan\theta$ we obtain:

$$\dot{\theta}(1 + \tan\theta^2) = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2} \quad (11)$$

So we have:

$$\dot{\theta} = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{\dot{x}^2 + \dot{y}^2} \quad (12)$$

The third equation of the system done:

$$\tan\varphi = \frac{l\dot\theta}{\vartheta} = l\frac{\ddot y\dot x - \dot y\ddot x}{(\dot x^2+\dot y^2)^{\frac{3}{2}}} \quad (13)$$

It is easy to see that all variables of the system denoting by $(x, y, \theta, u, \varphi)$ can be expressed in function of x and y and of their derivatives until the order 2, this result is compatible with the principle of flatness. The interpretation of the singularities at the speed equal to zero, is that the robot does not stop nor starts brutally, as well as if it turns, there is nothing. To overcome these singularities, we decouple the geometric aspect of the trajectory of evolution on this path. For this we will proceed by the parameterizations of the curvilinear abscissa $\sigma(t)$such that if T is the duration of movement, so we have: $\sigma(0) = 0, \sigma(T) = 1, \dot\sigma(0) = \dot\sigma(T) = 0, \sigma(t) = (\frac{t}{T})^2(3-2\frac{t}{T})$

Considering the geometry of the robot trajectory described by: $[x_d(\sigma), y_d(\sigma)]$and using the definition of the curvilinear abscissa:

$$\dot x^2 + \dot y^2 = 1 \quad (14)$$

So we obtain

$$u(t) = \dot\sigma(t) \quad (15)$$

$$\theta(t) = \arctan\frac{\dot y}{\dot x} \quad (16)$$

$$\varphi = \arctan l\frac{\ddot y\dot x - \dot y\ddot x}{(\dot x^2+\dot y^2)^{\frac{3}{2}}} \quad (17)$$

4 Flatness and linearization

We are interested in this paragraph to appear a dynamic endogenous feedback. In fact, we can put

$$x^2 = \nu_1 \quad (18)$$

$$y^2 = \nu_2 \quad (19)$$

Then, the dynamic endogenous feedback can be calculated by identifying the derivatives of$x$ and y until order 2 with their expressions in terms of inputs$u$ and φ. By deriving equations (1) and (2) and using (14) and (15), we have:

$$\ddot x = \dot u - \frac{u^2}{l}\sin\theta\tan\varphi = \nu_1 \quad (20)$$

$$\ddot y = \dot u + \frac{u^2}{l}\cos\theta\tan\varphi = \nu_2 \quad (21)$$

This trajectory $y = Y(x)$ must verify the four conditions denoting by:

$$y_i = Y(x_i), \frac{dY}{dx}(x_i) = 0, \frac{d^2Y}{d^2x}(x_i) = 0 \quad (22)$$

$$y_f = Y(x_f), \frac{dY}{dx}(x_f) = 0, \frac{d^2Y}{d^2x}(x_f) = 0 \quad (23)$$

We can choose the polynomial of third degree in x denoting by:

$$Y(x) = y_i + (y_f - y_i)(\frac{x-x_i}{x_f-x_i})^3(10 - 15(\frac{x-x_i}{x_f-x_i}) + 6(\frac{x-x_i}{x_f-x_i})^2) \quad (24)$$

which satisfied the last conditions. It remains to construct the trajectory $t \longrightarrow x(t)$ which verifies:

$$x(t_i) = x_i, \dot x(t_i) = 0 \quad (25)$$

$$x(t_f) = x_f, \dot x(t_f) = 0 \quad (26)$$

Then, we will get the following polynomial denoting by:

$$x(t) = x_i + (x_f - x_i)(\frac{t-t_i}{t_f-t_i}^2(3 - 2(\frac{t-t_i}{t_f-t_i})) \quad (27)$$

5 Flatness and tracking of trajectory

By using the expression of the dynamic endogenous feedback calculated in section 4 and the expressions of (19), (20), we will have the following curly system

$$\dot x = u\cos\theta \quad (28)$$

$$\dot y = u\sin\theta \quad (29)$$

$$\dot\theta = \frac{1}{u}(-\nu_1\sin\theta + \nu_2\cos\theta) \quad (30)$$

$$\dot\vartheta = \nu_1\cos\theta + \nu_2\sin\theta \quad (31)$$

We have a curly system of order 2 equal to the order of the curly system (17) and (18). Thus, we can choose, if we measure all the state (x, y, θ) and if $u \neq 0$, we obtain the following expressions corresponds of the new controls denoting by:

$$\vartheta_1 = \nu_1^* - \sum k_{1j}(x^{(j)} - x^*)^{(j)}) \quad (32)$$

$$\vartheta_2 = \nu_2^* - \sum k_{2j}(y^{(j)} - (y^*)^{(j)}) \quad (33)$$

ν_1^* and ν_2^* are the inputs of references which corresponds of trajectories of references x^* and y^* . The constant k_{1j} and k_{2j} are chosen in order to assure the stability of the systems denoting by:

$$e_1^{(2)} + k_{11}e_1^{(1)} + k_{10}e_1 = 0 \quad (34)$$

$$e_2^{(2)} + k_{21}e_2^{(1)} + k_{20}e_2 = 0 \quad (35)$$

6 Simulations

Figures 2 and 3 shows the movements of the real trajectory which try to follow the desired one along the

X axis despite the presence of disturbances. Figures 4 and5 represents the movements of the real trajectory which try also to follow the desired one along the Y axis in the presence of disturbances. The last figures presents the two controls of the vehicle which are in oscillation phase in order to force real trajectories to follow the desired one. Finally, it is obvious that the satisfactory output tracking performance has been almost achieved through the proposed control scheme.

7 Conclusion

In this paper we are proposed a flatness control of a vehicle in presence of disturbances. A flatness control is used to generate the desired trajectory and to force the vehicle to follow it. It is clear that the tracking errors resulting for the two movements (X axis and Y axis) are almost acceptable and the oscillation which present in the controls of the system are due to the disturbances, in this case we try to find another method more robust to perturbations which can force the system to follow trajectories without any oscillations at the controls.

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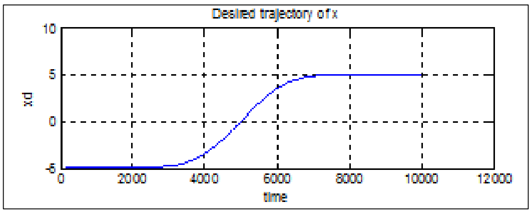

Figure 2: The desired trajectory of x

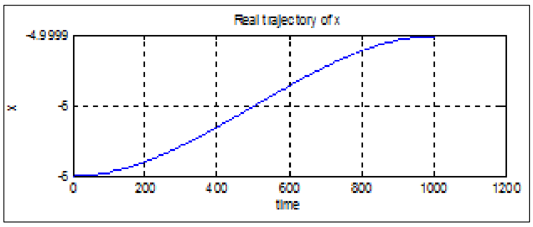

Figure 3: The real trajectory of x

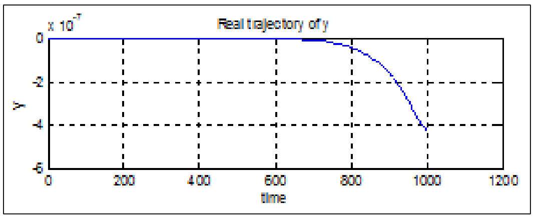

Figure 4: The real trajectory of y

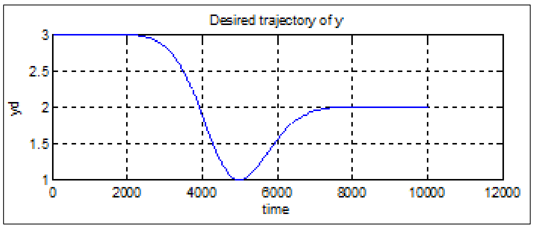

Figure 5: The desired trajectory of y

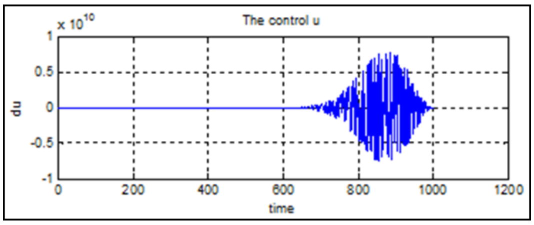

Figure 6: The control using flatness

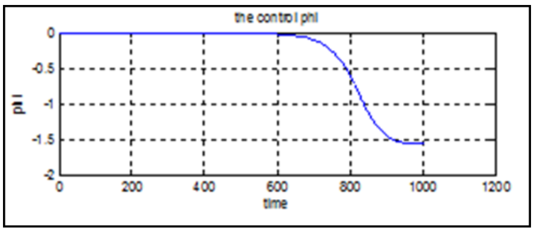

Figure 7: The second control using flatness